### Grade 5 Math Instructional Calendar

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<td>Use Models and Strategies to Divide Decimals (10 days)</td>
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<td>5.OA.2.3</td>
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<td>Geometric Measurement: Classify Two-Dimensional Figures (6 days)</td>
<td>5.G.2.3 5.G.2.4</td>
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<td>May 4 – 15 (FSA Window)</td>
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<td>May 25 (Memorial Day)</td>
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<td>May 29 (VMT 4)</td>
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# Unit 1

**PACING:** Aug. 12 - Oct. 11

## Topic 1: Understanding Place Value

**Standards**

- Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and \( \frac{1}{10} \) of what it represents in the place to its left.

**Academic Language**

- base-ten numeral
- digit
- equivalent decimals
- expanded form
- exponent
- multi-digit number names
- power of 10
- value
- whole

**Students will:**

- **demonstrate** with models that in a multi-digit number, a digit in one place represents ten times what it represents in the place to its right.

- **demonstrate** with models that a digit in one place represents \( \frac{1}{10} \) the value of what it represents in the place to its left.

- **explain** the relationship between the values of digits across multiple place values, using multiplicative comparison.

**Assessment Limit**

- Items may require a comparison of the values of digits across multiple place values, including whole numbers and decimals from millions to thousandths.

## Standards

<table>
<thead>
<tr>
<th>MAFS.5.NBT.1.1</th>
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</thead>
<tbody>
<tr>
<td>base-ten numeral</td>
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<tr>
<td>digit</td>
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<tr>
<td>equivalent decimals</td>
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<tr>
<td>expanded form</td>
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<tr>
<td>exponent</td>
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<tr>
<td>multi-digit number names</td>
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<tr>
<td>power of 10</td>
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<tr>
<td>value</td>
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<tr>
<td>whole</td>
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</tbody>
</table>

**Students will:**

- **express** powers of 10 using whole-number exponents.

**NOTE:** While students may **discover** a pattern of “adding zeros”, they need to be able to justify this pattern in terms of the number of times 10 is used as a factor (as denoted by the exponent).

E.g.,\(10^1 = 10\), \(10^2 = 10 \times 10 = 100\), \(10^3 = 10 \times 10 \times 10 = 1,000\)
- **explain** the pattern for how and why the number of zeros in a product (when multiplying a whole number by a power of 10) relates to the power of 10.
  E.g., \(5 \times 10^2 = 500\) because multiplying by \(10^2\) is the same as using 10 as a factor two times \((5 \times 10 \times 10)\) and every time a number is multiplied by 10 the digits shift one place to the left in the product, so multiplying by 10 two times shifts every digit 2 places to the left.

- **explain** the pattern in the placement of the decimal point when a decimal is multiplied by a power of 10 and how the placement of the decimal point relates to the power of 10.
  E.g., \(1.895 \times 10^3 = 1895\) because multiplying by \(10^3\) is the same as using 10 as a factor three times \((1.895 \times 10 \times 10 \times 10)\) and every time a number is multiplied by 10 the digits shift one place to the left in the product, so multiplying by 10 three times shifts every digit 3 places to the left, resulting in the decimal point in the product being 3 places to the right of where it is in the factor.

- **explain** the pattern in the placement of the decimal point when a decimal is divided by a power of 10 and how the placement of the decimal point relates to the power of 10.
  E.g., Every time a number is divided by 10, the digits shift one place to the right in the quotient, so when dividing by 10 two times such as in the expression \(15.3 \div 10^2\), every digit shifts 2 places to the right, resulting in the decimal point in the quotient being 2 places to the left of where it is in the dividend.

### Assessment Limit
Items may contain whole number and decimal place values from millions to thousandths.

Items may contain whole number exponents with bases of 10.

---

**Read, write, and compare decimals to thousandths.**

- **a.** Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., \(347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \left(\frac{1}{10}\right) + 9 \times \left(\frac{1}{100}\right) + 2 \times \left(\frac{1}{1000}\right)\).

- **b.** Compare two decimals to thousandths based on meanings of the digits in each place, using \(<\), \(=\), and \(>\) symbols to record the results of comparisons.

### Students will:

- **read and write** decimals from millions to thousandths using base-ten numerals, number names and expanded form.

  E.g., Some equivalent forms of 2.34 are:

  \[
  2 + 0.30 + 0.04 \\
  2 \times (1) + 3 \times (\frac{1}{10}) + 4 \times (\frac{1}{100}) \\
  2 \times (1) + 3 \times (\frac{1}{10}) + 4 \times (\frac{1}{100}) + 0 \times (\frac{1}{1000}) \\
  2 \times (1) + 34 \times (0.01) \\
  2 \times (1) + 34 \times (\frac{1}{100})
  \]

- **compare** two decimals from millions to thousandths using place value and record the comparison using symbols <, >, or =.

### Assessment Limit
Items may contain decimals to the thousandths with the greatest place value to the millions.
Use place value understanding to round decimals to any place

**Students will:**
- use place value to round decimals to any place.

E.g.,

**NOTE:** When rounding decimal numbers, the final place value in the rounded number indicates where the number was rounded to; E.g., When rounding 34.632 to the nearest tenth, a result of 34.600 suggests that the number has been rounded to the thousandths place so, in order to attend to precision, the two right-most zeros should be omitted.

**Assessment Limit**
Items may contain decimals to the thousandths with the greatest place value to the millions. The least place value a decimal may be rounded to is the hundredths place.

**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

**Applicable information from the progression documents:**

**Understand the place value system**
Students extend their understanding of the base-ten system to the relationship between adjacent places, how numbers compare, and how numbers round for decimals to thousandths.
New at Grade 5 is the use of whole number exponents to denote powers of 10. Students understand why multiplying by a power of 10 shifts the digits of a whole number or decimal that many places to the left. For example, multiplying by $10^4$ is multiplying by 10 four times. Multiplying by 10 once shifts every digit of the multiplicand one place to the left in the product (the product is ten times as large) because in the base-ten system the value of each place is 10 times the value of the place to its right. So multiplying by 10 four times shifts every digit 4 places to the left. Patterns in the number of 0s in products of a whole number and a power of 10 can be explained in terms of place value. Because students have developed their understandings of and computations with decimals in terms of multiples (consistent with 4.OA.4) rather than powers, connecting the terminology of multiples with that of powers affords connections between understanding of multiplication and exponentiation.

(See p. 18 in the NBT Progressions.)
## Topic 2: Use Models and Strategies to Add and Subtract Decimals

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
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<tbody>
<tr>
<td>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td>MAFS.5.NBT.2.7</td>
</tr>
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</table>

### Students will:
- **add** up to three decimals to hundredths, using concrete models, drawings, strategies based on place value, and/or properties of operations.
- **subtract** decimals to hundredths, using concrete models, drawings, strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
- **represent and explain** strategies and reasoning used to solve addition and subtraction problems including decimals.

**E.g., Use a model to solve 3 – 0.6**

![Model for 3 - 0.6](image1)

**E.g., Use a model to solve 0.3 + 0.25**

![Model for 0.3 + 0.25](image2)

### NOTE:
This standard requires students to extend the models and strategies previously developed for conceptual understanding of operations with whole numbers. Using the standard algorithms for adding, subtracting, multiplying, and dividing decimals is a Grade 6 standard.

### NOTE:
Encourage students to use mental math, to assess the reasonableness of answers, and to round to estimate throughout the lessons in the topic, not just in isolated places.

### Assessment Limit
Items may only use factors that result in decimal solutions to the thousandths place (e.g., multiplying tenths by hundredths). Items may not include multiple different operations within the same expression (e.g., $21 + 0.34 \times 8.55$). Expressions may have up to two procedural steps of the same operation.
<table>
<thead>
<tr>
<th>Aspects of Rigor targeted by the standards in this topic:</th>
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<tbody>
<tr>
<td>• <strong>Conceptual understanding:</strong> The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.</td>
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<table>
<thead>
<tr>
<th>Applicable information from the progression document:</th>
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<tr>
<td>Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers. Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points).</td>
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<td>(See p. 19 in the NBT Progressions.)</td>
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### Topic 3: Fluently Multiply Multi-Digit Whole Numbers

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<td>Fluently multiply multi-digit whole numbers using the standard algorithm.</td>
<td></td>
<td>factor product standard algorithm</td>
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#### Students will:
- **apply** an understanding of multiplication, place value, and flexibility with multiple strategies to use the standard algorithm for multiplication.

**NOTE:** Computational fluency is defined as accuracy, efficiency, and flexibility. Grade 5 is the first grade-level in which students are expected to be fluent with the standard algorithm for multiplication.

**NOTE:** Encourage students to use mental math, to assess the reasonableness of answers, and to round to estimate throughout the lessons in the topic.

**NOTE:** Throughout the topic, teachers should consistently have students connect their previous understandings of multiplication and the strategies they used in grade 4 (4.NBT.2.5) to the standard algorithm.

<table>
<thead>
<tr>
<th>Assessment Limit</th>
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<tr>
<td>Multiplication may not exceed five digits by two digits.</td>
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<th>Aspects of Rigor targeted by the standards in this topic:</th>
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<tbody>
<tr>
<td>• <strong>Procedural skill and fluency:</strong> The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.</td>
</tr>
</tbody>
</table>

**Applicable information from the progression document:**

At Grade 5, students fluently compute products of whole numbers using the standard algorithm. Underlying this algorithm are the properties of operations and the base-ten system.

(See p. 18 in the NBT Progressions.)
**Topic 4:** Use Models and Strategies to Multiply Decimals

**Standards**

Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**MAFS.5.NBT.2.7**

**Academic Language**

- **area model**
- **factor**
- **partial product**
- **product**

**Students will:**

- **multiply** decimals using rectangular arrays, area models, and/or an understanding of place value and properties of operations.
- **represent and explain** strategies and reasoning used to solve multiplication problems including decimals.

**E.g.,** Use a model to solve $0.6 \times 1.2 = 0.72$

![Rectangular array for multiplication](image)

**E.g.,** Use a model to solve $1.5 \times 1.7 = 2.55$

![Area model for multiplication](image)
NOTE: This standard requires students to extend the models and strategies previously developed for conceptual understanding of operations with whole numbers. Using the standard algorithms for adding, subtracting, multiplying, and dividing decimals is a Grade 6 standard.

NOTE: Students should not be required to use a specific model or strategy in each lesson, but rather emphasize using concrete models or drawings and place value strategies throughout. Use these lessons as an opportunity for students to make sense of the strategies and apply them to appropriate multiplication problems. Students should be able to relate their strategies to written methods and explain their reasoning.

Assessment Limit
Items may only use factors that result in decimal solutions to the thousandths place (e.g., multiplying tenths by hundredths).
Items may not include multiple different operations within the same expression (e.g., $21 + 0.34 \times 8.55$).
Expressions may have up to two procedural steps of the same operation.

Aspects of Rigor targeted by the standards in this topic:
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
- **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.

Applicable information from the progression document:
General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0.1 and by 0.01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0.1 and 0.01 (e.g., 0.2 and 0.02, etc.). (See p. 19 in the NBT Progressions.)
# Unit 2

## Topic 5: Use Models and Strategies to Divide Whole Numbers

### Pacing: Oct. 15 – Jan. 27

### Standards

Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**Students will:**

- solve division of up to a four-digit dividend by a two-digit divisor using strategies based on place value, properties of operations, and/or the relationship between multiplication and division.
- illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

**E.g.,**

\[ 460 \div 12 = 40 \]

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<th>10</th>
<th>10</th>
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<tbody>
<tr>
<td>12</td>
<td>2</td>
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</tbody>
</table>

### Academic Language

- area model
- dividend
- divisor
- factor
- groups of partial quotient
- quotient
- rectangular array

**MAFS.5.NBT.2.6**

**Assessment Limit**

Division may not exceed four digits by two digits.

**NOTE:** Use of the standard algorithm for division is a Grade 6 standard.

**NOTE:** 5.NBT.2.6 calls for students to illustrate and explain calculations by using equations, rectangular arrays, and/or area models. Partial Quotients is an appropriate strategy for 5.NBT.2.6, as it is a strategy based on place value, however it is not specifically named in the standard as one of the ways grade 5 students must illustrate and explain calculations.
Aspects of Rigor targeted by the standards in this topic:

- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

Applicable information from the progression document:

Division in Grade 5 extends Grade 4 methods to two-digit divisors. Students continue to decompose the dividend into base-ten units and find the quotient place by place, starting from the highest place. They illustrate and explain their calculations using equations, rectangular arrays, and/or area models. Estimating the quotients is a new aspect of dividing by a two-digit number. Even if students round the dividend appropriately, the resulting estimate may need to be adjusted up or down. Sometimes multiplying the ones of a two-digit divisor composes a new thousand, hundred, or ten. These newly composed units can be written as part of the division computation, added mentally, or as part of a separate multiplication computation. Students who need to write decomposed units when subtracting need to remember to leave space to do so.

(See p. 18 in the NBT Progressions.)
### Topic 6: Use Models and Strategies to Divide Decimals

<table>
<thead>
<tr>
<th>Standards</th>
<th>Pacing: Nov. 1 - 15</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.</td>
<td><strong>MAFS.5.NBT.2.7</strong></td>
<td>area model, dividend, divisor, factor, groups of, partial quotient, quotient, rectangular array</td>
</tr>
</tbody>
</table>

**Students will:**
- divide decimals using rectangular arrays, area models, an understanding of place value and properties of operations, and/or the relationship between multiplication and division.
- represent and explain strategies and reasoning used to solve problems including decimals.

**E.g.,**

![Diagram of dividing decimals using rectangular arrays and area models](image)

Think multiplication:
To find 3.60 ÷ 1.20, use the relationship between multiplication and division.

\[1.20 \times ? = 3.60\]

Writing this another way:
120 hundredths \(\times ? = 360\) hundredths

\[? = 3\]

**NOTE:** This standard requires students to extend the models and strategies previously developed for conceptual understanding of operations with whole numbers. Using the standard algorithms for adding, subtracting, multiplying, and dividing decimals is a Grade 6 standard.

**Assessment Limits**
Items may not include multiple different operations within the same expression (e.g., \(21 + 0.34 \times 8.55\)).
Expressions may have up to two procedural steps of the same operation.

**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
Applicable information from the progression document:

General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient.

As with decimal multiplication, students can first examine the cases of dividing by 0.1 and 0.01 to see that the quotient becomes 10 times or 100 times as large as the dividend (see also the Number and Operations—Fractions Progression). For example, students can view 70/0.11 as asking how many tenths are in 7. Because it takes 10 tenths to make 1, it takes 7 times (5.NF.7b) as many tenths to make 7, so 70/0.1 = 70.

(5.NF.7b Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. b Interpret division of a whole number by a unit fraction, and compute such quotients.)

Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, 7 0.1s is the same as 70 1s. So dividing by 0.1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate 70/0.2, students can reason that 0.2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 0.2s is the same as 70 2s; multiplying both the 7 and the 0.2 by 10 results in the same quotient. Or students could calculate 7 0.2s by viewing 0.2 as 2 0.1s, so they can first divide 7 by 2, which is 3.5, and then divide that result by 0.1, which makes 3.5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend (5.NF.5) and moves the digits of the dividend one place to the left.

5.NF.5 Interpret multiplication as scaling (resizing), by:
   a) Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
   b) Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence to the effect of multiplying ab by 1.
### Topic 7: Use equivalent fractions to add and subtract fractions

<table>
<thead>
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<th>Standards</th>
<th>Pacing: Nov. 25 – Dec. 18</th>
<th>Academic Language</th>
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<tbody>
<tr>
<td>Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$ (In general, $\frac{a}{b} + \frac{c}{d} = \frac{(ad + bc)}{bd}$.)</td>
<td>MAFS.5.NF.1.1</td>
<td>denominator difference equivalent fraction greater than 1 mixed number numerator sum</td>
</tr>
</tbody>
</table>

**Students will:**
- represent addition and subtraction of fractions, including mixed numbers and fractions greater than 1, with unlike denominators using concrete models and representations.
- apply concepts of equivalent fractions, and decomposition of fractions to find like denominators to add and/or subtract.

E.g.,

\[
\begin{align*}
\frac{1}{4} + \frac{2}{3} & \quad \downarrow \\
\frac{1 \times 3}{4 \times 3} + \frac{2 \times 4}{3 \times 4} & = \frac{11}{12} \\
\frac{3}{12} + \frac{8}{12} & = \frac{11}{12}
\end{align*}
\]

**NOTE:** Throughout the topic, make explicit connections between the work of grade 4 around generating equivalent fractions to support the grade 5 work of adding and subtracting fractions with unlike denominators. Students should also be presented with numerous opportunities to recognize that sums and differences may be represented as equivalent fractions, rather than requiring students to “simplify”, “reduce”, or find the “lowest terms”.

**NOTE:** Identify and present opportunities for students to add two fractions less than 1 that result in a sum greater than 1 and also to subtract fractions less than 1 and greater than 1 from a whole number in order to allow for full scope of numbers expected by 5.NF.1.1 and 5.NF.1.2.
### Assessment Limit
Fractions greater than 1 and mixed numbers may be included.
Expressions may have up to three terms.
Least common denominator is not necessary to calculate sums or differences of fractions.
Items may not use the terms “simplify” or “lowest terms.”
For given fractions in items, denominators are limited to 1-20.
Items may require the use of equivalent fractions to find a missing term or part of a term.

### MAFS.5.NF.1.2

**Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers.** For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

**Students will:**
- solve word problems involving addition and subtraction of fractions with like and unlike denominators.
- use benchmark fractions and number sense of fractions to estimate and assess reasonableness of answers.

E.g., At Lynnette’s birthday party, the boys ate $\frac{3}{5}$ of a party sub and the girls at the party ate $\frac{9}{12}$ of the same-size sub. Without actually adding these fractions, can you tell if more or less than a whole party sub was eaten?

The student is able to correctly explain that more than one sub was eaten because both fractions are greater than $\frac{1}{2}$, so, their sum must be more than a whole.

**NOTE:** See the Common Addition and Subtraction Situations Table for examples of word problem types (p.47).

### Assessment Limit
Fractions greater than 1 and mixed numbers may be included.
Expressions may have up to three terms.
Least common denominator is not necessary to calculate sums or differences of fractions.
Items may not use the terms "simplify" or “lowest terms.”
For given fractions in items, denominators are limited to 1-20.
Items may require the use of equivalent fractions to find a missing term or part of a term.

### Aspects of Rigor targeted by the standards in this topic:
- **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.
- **Application:** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.
Applicable information from the progression document:

In Grade 4, students have some experience calculating sums of fractions with different denominators...where one denominator is a divisor of the other, so that only one fraction has to be changed. Grade 5 students extend this reasoning to situations where it is necessary to re-express both fractions in terms of a new denominator. For example, in calculating $\frac{2}{3} + \frac{5}{4}$ they reason that if each third in $\frac{2}{3}$ is subdivided into fourths, and if each fourth in $\frac{5}{4}$ is subdivided into thirds, then each fraction will be a sum of unit fractions with denominator $3 \times 4 = 4 \times 3 = 12$:

$$\frac{2}{3} + \frac{5}{4} = \frac{2\times4}{3\times4} + \frac{5\times3}{4\times3} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$$

Students make sense of fractional quantities when solving word problems, estimating answers mentally to see if they make sense. For example in the problem

Ludmilla and Lazarus each have a lemon. They need a cup of lemon juice to make hummus for a party. Ludmilla squeezes $\frac{1}{2}$ a cup from hers and Lazarus squeezes $\frac{3}{5}$ of a cup from his. How much lemon juice do they have? Is it enough?

Students estimate that there is almost but not quite one cup of lemon juice, because $\frac{3}{5} < \frac{1}{2}$. They calculate $\frac{1}{2} + \frac{2}{5} = \frac{9}{10}$, and see this as less than 1, which is probably a small enough shortfall that it will not ruin a recipe.

(See p. 11 in the NF Progressions)
## Topic 8: Apply Understanding of Multiplication to Multiply Fractions

### Standards

Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

- **a.** Interpret the product \((\frac{a}{b}) \times q\) as a parts of a partition of \(q\) into \(b\) equal parts; equivalently, as the result of a sequence of operations \(a \times q \div b\). For example, use a visual fraction model to show \((\frac{2}{3}) \times 4 = \frac{8}{3}\); and create a story context for this equation. Do the same with \((\frac{2}{3}) \times \frac{4}{5} = \frac{8}{15}\). (In general, \((\frac{a}{b}) \times (\frac{c}{d}) = \frac{ac}{bd}\).)

- **b.** Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

### Academic Language

- area
- denominator
- equivalent
- fraction
- factor
- fraction greater than 1
- mixed number
- numerator
- partition
- product
- rectangular array
- scale factor
- unit fraction

### Students will:

- **use** visual fraction models (e.g., fraction strips, arrays, area models, fraction multipliers, number lines, etc.) to interpret multiplication of a whole number by a fraction.
  
  E.g., \(\frac{2}{3}\) of 5 (i.e., \(\frac{2}{3} \times 5\)) is 2 parts when 5 is partitioned into 3 equal parts.

- **use** visual fraction models (e.g., fraction strips, arrays, area models, fraction multipliers, number lines, etc.) to interpret multiplication of a fraction by a fraction.
  
  E.g., Extend the reasoning that since \(\frac{2}{3}\) of 5 is 2 parts when 5 is partitioned into 3 equal parts to understand that \(\frac{2}{3} \times \frac{5}{6}\) (i.e., \(\frac{2}{3} \times \frac{5}{6}\)) is 2 parts when \(\frac{5}{6}\) is partitioned into 3 equal parts.

- **create** story contexts for problems involving multiplication of a whole number by a fraction or multiplication of two fractions.

- **use** unit squares with fractional sides to find the area of a rectangle with fractional side lengths by tiling it, and use this tiling to prove that the area is the same as would be found by multiplying the side lengths.

- **multiply** fractional side lengths to find areas of rectangles.

- **represent** fraction products as rectangular areas.

E.g., \(\frac{2}{3} \times \frac{4}{5}\) represented using an area model that measures 1 unit by 1 unit:
Assessment Limit
Visual models may include:
• Any appropriate fraction model (e.g., circles, tape diagrams, polygons, etc.)
• Rectangle models tiled with unit squares
For tiling, the dimensions of the tile must be unit fractions with the same denominator as the given rectangular shape.
Items may not use the terms "simplify" or "lowest terms."
Items may require students to interpret the context to determine operations.
Fractions may be greater than 1.
For given fractions in items, denominators are limited to 1-20.

Interpret multiplication as scaling (resizing), by:
  a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
  b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence \( \frac{a}{b} = \left( \frac{n}{n} \times \frac{a}{n} \right) \) to the effect of multiplying \( \frac{a}{b} \) by 1.

Students will:
• interpret the relationship between the size of the factors and the size of the product without performing the actual multiplication.

NOTE: Whole number factors should be greater than 1,000.

E.g.,

Example 1:
How does the product of 3,225 x 6,000 compare to the product of 3,225 x 3,000?
How do you know?
Since 3,000 is half of 6,000, the product of 3,225 x 6000 will be double or twice the product of 3,225 x 3000.

• explain why multiplying a given number by a scale factor greater than 1 (i.e., a whole number, a fraction greater than 1, or a mixed number) results in a product greater in size than the given number.
• explain why multiplying a given number by a scale factor less than 1 results in a product less in size than the given number.
• explain why multiplying a given fraction by a fraction that is equivalent to 1 as a scale factor results in a product that is equivalent in size to the given fraction and use this understanding to find equivalent fractions.

E.g., \( \frac{7}{9} \times 1 = \frac{7}{9} \times \frac{4}{4} = \frac{28}{36} \)

Example 2:
Two newspapers are comparing sales from last year.
  o The Post sold 34,859 copies.
  o The Tribune sold one-and-a-half times as many copies as the Post.
Write an expression which describes the number of newspapers the Tribune sold.
\[ 1\frac{1}{2} \times 34,859 \]
Assessment limit
For given fractions in items, denominators are limited to 1-20.
Non-fraction factors in items must be greater than 1,000.
Scaling geometric figures may not be assessed.
Scaling quantities of any kind in two dimensions is beyond the scope of this standard.

Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem.  

Students will:
- solve real world problems involving multiplication of fractions and mixed numbers (including multiplying mixed numbers by mixed numbers).

Aspects of Rigor targeted by the standards in this topic:
- Conceptual understanding: The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
- Procedural skill and fluency: The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.
- Application: The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

Applicable information from the progression document:

Using a number line to show that $\frac{2}{3} \times \frac{5}{2} = \frac{2 \times 5}{3 \times 2}$

Using a fraction strip to show that $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
In preparation for Grade 6 work in ratios and proportional relationships, students learn to see products such as 5 x 3 or 1/2 x 3 as expressions that can be interpreted in terms of a quantity, 3, and a scaling factor, 5 or 1/2. Thus, in addition to knowing that 5 x 3 = 15, they can also say that 5 x 3 is 5 times as big as 3, without evaluating the product. Likewise, they see 1/2 x 3 as half the size of 3.

Grade 5 work with multiplying by unit fractions, and interpreting fractions in terms of division, enables students to see that multiplying a quantity by a number smaller than 1 produces a smaller quantity, as when the budget of a large state university is multiplied by 1/2, for example. (See pp. 12–13 in the NF Progressions.)
# Unit 3

**PACING:** Jan. 28 – Mar. 27

## Topic 9: Apply Understanding of Division to Divide Fractions

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interpret a fraction as division of the numerator by the denominator ( \frac{a}{b} = a \div b ). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. <em>For example, interpret ( \frac{2}{3} ) as the result of dividing 3 by 4, noting that ( \frac{2}{3} ) multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size ( \frac{3}{4} ). If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?</em></td>
<td>( \text{MAFS.5.NF.2.3} )</td>
</tr>
</tbody>
</table>

**Students will:**

- **illustrate** that the numerator represents the total amount being divided (dividend) and that the denominator represents the number of equal portions needed (divisor).
- **explain** that fractions \( \frac{a}{b} \) can be represented as division of the numerator by the denominator \( a \div b \). For example, \( \frac{5}{3} = 5 \div 3 \).
- **solve** word problems involving the division of whole numbers resulting in a fractional or mixed number quotient.

E.g.,

Sara has 3 sub sandwiches. She would like to split the sandwiches equally between 7 people. What fraction of 1 sandwich will each person receive?

- Divide each of 3 rectangles into 7 equal parts resulting in a total of 21 one-seventh sized pieces.
- Divide the 21 one-seventh sized pieces into 7 equal groups.

\[
\begin{array}{cccccccc}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
1 & 1 & 2 & 2 & 4 & 4 & 5 & 5 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
1 & 1 & 6 & 6 & 6 & 7 & 7 & 7 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
1 & 1 & 7 & 7 & 7 & 7 & 7 & 7 \\
7 & 7 & 7 & 7 & 7 & 7 & 7 & 7 \\
\end{array}
\]

- The result is 3 one-seventh sized pieces per group. \( 3 \times \frac{1}{7} = \frac{3}{7} \). So, \( 3 \div 7 = \frac{3}{7} \).

**NOTE:** See the *Common Multiplication and Division Situations Table* for examples of division word problem types (p.48).

## Assessment Limit

- Quotients in division items may not be equivalent to a whole number.
- Items may contain fractions greater than 1.
- Items may not use the terms “simplify” or “lowest terms.”
- Only use whole numbers for the divisor and dividend of a fraction.
- For given fractions in items, denominators are limited to 1-20.
Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.)

<table>
<thead>
<tr>
<th>a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for $\frac{1}{3} \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $\frac{1}{3} \div 4 = \frac{1}{12}$ because $\frac{1}{12} \times 4 = \frac{1}{3}$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for $4 \div \frac{1}{3}$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div \frac{1}{3} = 20$ because $20 \times \frac{1}{3} = 4$.</td>
</tr>
<tr>
<td>c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$ lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins?</td>
</tr>
</tbody>
</table>

**Students will:**

- **apply** an understanding of the division of whole numbers to the concept of dividing whole numbers by unit fractions (e.g., $4 + \frac{1}{3}$ can be interpreted as 4 inches of ribbon being cut into $\frac{1}{3}$-inch pieces) and dividing unit fractions by whole numbers (e.g., $\frac{1}{3} + 4$ can be interpreted as sharing $\frac{1}{3}$ of a large pizza with 4 people).

- **interpret and solve** problems involving the division of whole numbers by unit fractions (e.g., $4 + \frac{1}{3}$ can be thought of as how many groups of $\frac{1}{3}$ can be made from 4 wholes) using visual models and story contexts.

E.g., How many quarter cups are in 2 cups?

2 $\div \frac{1}{4} = 8$

There are 8 quarter cups in 2 cups.

- **interpret and solve** problems involving the division of unit fractions by whole numbers (e.g., $\frac{1}{3} + 4$ can be thought of as $\frac{1}{3}$ being divided into 4 equal parts) using visual models and story contexts.
E.g., \( \frac{1}{3} \div 4 = ? \)

![Fraction representation](image)

By dividing each third into 4 equal parts, we show that each part is \( \frac{1}{12} \) of the whole. This means that \( \frac{1}{3} \div 4 = \frac{1}{12} \).

- **solve** real world problems involving division of unit fractions and whole numbers using fraction models and equations.

**NOTE:** Division of a fraction by a fraction is a Grade 6 standard.

**NOTE:** Allow opportunities for students to create context for whole number divided by unit fraction and unit fraction divided by whole number.

**Assessment limit**
For given fractions in items, denominators are limited to 1-20

**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding**: The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

- **Application**: The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

**Applicable information from the progression documents:**
In Grade 5, [students] connect fractions with division, understanding that \( 5 \div 3 = 5/3 \), or, more generally, \( a/b = a \div b \) for whole numbers \( a \) and \( b \), with \( b \) not equal to zero.
In Grade 5, they connect fractions with division, understanding that \( \frac{5}{3} = 5 \times \frac{1}{3} = \frac{5}{3} \), or, more generally, \( \frac{a}{b} = a \div b \) for whole numbers \( a \) and \( b \), with \( b \) not equal to zero. They can explain this by working with their understanding of division as equal sharing (see figure in margin). They also create story contexts to represent problems involving division of whole numbers. For example, they see that:

If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get?

can be solved in two ways. First, they might partition each pound among the 9 people, so that each person gets \( 50 \times \frac{1}{9} = \frac{50}{9} \) pounds. Second, they might use the equation \( 9 \times 5 = 45 \) to see that each person can be given 5 pounds, with 5 pounds remaining. Partitioning the remainder gives \( \frac{5}{9} \) pounds for each person.

(See p. 12 in the NF Progressions.)
### Topic 10: Represent and Interpret Data

<table>
<thead>
<tr>
<th>Standards</th>
<th>Pacing: Feb. 11 - 21</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a line plot to display a data set of measurements in fractions of a unit (\frac{1}{2}, \frac{1}{4}, \frac{1}{8}). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.</td>
<td>MAFS.5.MD.2.2</td>
<td>data set, line plot, number line</td>
</tr>
</tbody>
</table>

**Students will:**
- *create* a line plot recording measurement data including fraction units of halves, quarters, and eighths.
- *use* the measurement data on a line plot to solve multistep problems involving fractions and draw conclusions about the data.

**NOTE:** Since students worked with categorical data and bar graphs in Grades 2 & 3, a student might find it natural to summarize a measurement data set by viewing it in terms of categories—the categories in question being the three distinct length values which appear on the number line above. For example, the student might want to say that there are two observations in the “category” of \(8\frac{1}{4}\) inches. However, it is important to recognize that \(8\frac{1}{4}\) inches is not a category like “blue, yellow or red” Unlike these colors, \(8\frac{1}{4}\) inches is a numerical value with a measurement unit. That difference is why the data in this table are called measurement data and presented on a line plot rather than a bar graph. A display of measurement data must present the measured values with their appropriate magnitudes and spacing on the number line of the line plot.

**NOTE:** On a number line, each tick mark should be labeled with the value that represents that tick mark’s distance from 0. Therefore, when labeling number lines that include units greater than 1, tick marks should be labeled with the mixed number that represents each point’s location (e.g., rather than label the tick marks after 8 on the number line above as \(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\), they are labeled precisely as \(8\frac{1}{4}, 8\frac{1}{2}, 8\frac{3}{4}\)).

**NOTE:** Data points on a line plot may be represented with an X or a dot.

**Assessment Limit**
Items requiring operations on fractions must adhere to the Assessment Limits for that operation’s corresponding standard.
### Aspects of Rigor targeted by the standards in this topic:

- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

- **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.

- **Application:** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

### Applicable information from the progression document:

Grade 5 students grow in their skill and understanding of fraction arithmetic, including multiplying a fraction by a fraction (5.NF.4), dividing a unit fraction by a whole number or a whole number by a unit fraction (4.NF.7), and adding and subtracting fractions with unlike denominators (5.NF.1). Students can use these skills to solve problems (5.NF.2, 5.NF.6, 5.NF.7c), including problems that arise from analyzing line plots. For example, given five graduated cylinders with different measures of liquid in each, students might find the amount of liquid each cylinder would contain if the total amount in all the cylinders were redistributed equally. (Students in Grade 6 will view the answer to this question as the mean value for the data set in question.)

(See p.11 in the MD Progressions.)
### Topic 11: Understand Volume Concepts

<table>
<thead>
<tr>
<th>Standards</th>
<th>Pacing: Feb. 24 – Mar. 6</th>
</tr>
</thead>
</table>
| Recognize volume as an attribute of solid figures and understand concepts of volume measurement.  
  a. A cube with side length 1 unit, called a “unit cube,” is said to have “one cubic unit” of volume, and can be used to measure volume.  
  b. A solid figure which can be packed without gaps or overlaps using \( n \) unit cubes is said to have a volume of \( n \) cubic units | MAFS.5.MD.3.3 |

**Students will:**
- identify volume as an attribute of a solid figure.  
- explain that a cube with 1 unit side lengths is “one cubic unit” of volume, and understand that this cubic unit is used to measure volume.  
- explain that the volume of a solid figure can be found by filling it with unit cubes, without gaps and overlaps, and finding the total number of unit cubes.

**Assessment limit**
Items may contain right rectangular prisms with whole-number side lengths. Figures may only be shown with unit cubes. Labels may include cubic units (i.e. cubic centimeters, cubic feet, etc.) or exponential units (i.e., \( \text{cm}^3 \), \( \text{ft}^3 \), etc.). Items requiring measurement of volume by counting unit cubes must provide a key of the cubic unit.

Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.  

**Students will:**
- measure the volume of a three-dimensional figure (i.e., rectangular prism and cube) by filling it with unit cubes and counting the number of unit cubes.

**E.g.**

![Diagram showing how to measure volume](image)

**NOTE:** Items may contain right rectangular prisms with whole-number side lengths. Figures may only be shown with unit cubes.
Relate volume to the operations of multiplication and addition and solve real-world and mathematical problems involving volume.

a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

b. Apply the formula $V = l \times w \times h$ and $V = B \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real-world and mathematical problems.

c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of non-overlapping parts, applying this technique to solve real-world problems.

**MAFS.5.MD.3.5**

**Students will:**

- **find** the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths or multiplying the height by the area of the base.

  E.g.,

  
  \[
  (3 \times 2) \text{ represents the number of cubic units on the first layer} \\
  (3 \times 2) \times 5 \text{ represents the number of } 3 \times 2 \text{ layers} \\
  (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) + (3 \times 2) \text{ represents } 5 \text{ layers with } 6 \text{ cubic units each} \\
  5 \times 6 = 30 \text{ cubic units}
  \]

- **relate** finding the product of three numbers (length, width, and height) using the associative property of multiplication to finding volume.

- **calculate** volume of rectangular prisms and cubes, with whole-number edge lengths, using the formula for volume ($V = l \times w \times h$ or $V = B \times h$) in real-world and mathematical problems.

  E.g.,

  When shown an image of a rectangular prism with labeled side lengths (no cubes shown), students may choose to use the formula $V = B \times h$:

  1. Find the area of the base by multiplying its length by its width ($B = l \times w$).
  2. Multiply the area of the base by the height ($V = B \times h$).
• **recognize** that volume is additive by decomposing a composite solid into non-overlapping rectangular prisms to **find** the volume of the solid by finding the sum of the volumes of each of the decomposed prisms.

E.g.,

```
3 layers with
8 cubic units in each layer
3 x 8 = 24
```

```
24 cubic units + 8 cubic units
Total Volume = 32 cubic units
```

• **solve** real world problems involving **finding** the volume of solid figures composed of two non-overlapping right rectangular prisms.

E.g., What is the volume of water needed to fill the pool in the diagram?

```
10 ft.
5 ft.
14 ft.
```

```
The deep end of the pool measures 14 ft. x 10 ft. x 5 ft. making the volume 700 ft³.
The shallow end of the pool measures 6 ft. x 5 ft. x 5 ft. making the volume 150 ft³.
700 ft³ + 150 ft³ = 850 ft³
```

**NOTE:** Labels may include cubic units (e.g., cubic centimeters, cubic feet, etc.) or exponential units (e.g., cm³, ft³, etc.).

**Assessment limit**

- Items may not contain fraction or decimal dimensions or volumes.
- Items may contain no more than two non-overlapping prisms – non-overlapping means that two prisms may share a face, but they do not share the same volume.
- Items assessing MAFS.5.MD.3.5b may not contain the use or graphic of unit cubes.
- Items assessing MAFS.5.MD.3.5c must contain a graphic of the figures.

**Aspects of Rigor targeted by the standards in this topic:**
### Conceptual understanding:
The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

### Procedural skill and fluency:
The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.

### Application:
The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

**Applicable information from the progression documents:**

“Packing” volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube (5.MD.C.3). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.MD.C.4).

Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism (5.MD.C.5).

(See p. 26 in the MD Progressions.)

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**Topic 12: Convert Measurements**

<p>| Pacing: Mar. 9 - 27 |</p>
<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convert among different-sized standard measurement units (i.e., km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec) within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems. <strong>NOTE:</strong> This standard has been amended in Florida to include specific units of measure.</td>
<td>MAFS.5.MD.1.1 conversion convert customary units FSA Reference Sheet larger unit metric units smaller unit</td>
</tr>
</tbody>
</table>

**Students will:**

- **convert** units (required units are listed on Grade 5 FSA Mathematics Reference Sheet) within the same system (customary to customary and metric to metric).
- **solve** multi-step word problems using measurement conversions.

**NOTE:** See the *Common Addition and Subtraction Situations Table* and the *Common Multiplication and Division Situations Table* for examples of word problem types (p.47 and 48).

### Assessment limit

Measurement values may be whole, decimal, or fractional values.
Conversions must be within the same system.

### Aspects of Rigor targeted by the standards in this topic:

- **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.

- **Application:** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

### Applicable information from the progression document:

“Packing” volume is more difficult than iterating a unit to measure length and measuring area by tiling. Students learn about a unit of volume, such as a cube with a side length of 1 unit, called a unit cube (5.MD.C.3). They pack cubes (without gaps) into right rectangular prisms and count the cubes to determine the volume or build right rectangular prisms from cubes and see the layers as they build (5.MD.C.4).

Students understand that multiplying the length times the width of a right rectangular prism can be viewed as determining how many cubes would be in each layer if the prism were packed with or built up from unit cubes. They also learn that the height of the prism tells how many layers would fit in the prism (5.MD.C.5).

(See p. 26 in the MD Progressions.)

Convert like measurement units within a given measurement system; in Grade 5, students extend their abilities from Grade 4 to express measurements in...
larger or smaller units within a measurement system (4.MD.1, 5.MD.1). This is an excellent opportunity to reinforce notions of place value for whole numbers and decimals, and connection between fractions and decimals (e.g., $2 \frac{1}{2}$ meters can be expressed as 2.5 meters or 250 centimeters). For example, building on the table from Grade 4, Grade 5 students might complete a table of equivalent measurements in feet and inches. Grade 5 students also learn and use such conversions in solving multi-step, real world problems (see example below).
## Unit 4

### PACING: Mar. 30 – May 29

#### Topic 13: Write and Interpret Numerical Expressions

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.</td>
<td>MAFS.5.OA.1.1</td>
</tr>
</tbody>
</table>

**Students will:**
- **evaluate** expressions which include parentheses, brackets, or braces
  
  E.g.,
  
<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 \times [5 + 3.5]</td>
<td>12 \times 8.5</td>
</tr>
<tr>
<td>7 \times 6 \times 2 \times 100</td>
<td>(7 \times 6) \times (2 \times 100)</td>
</tr>
<tr>
<td>5 \times (6 + 3 + 4)</td>
<td>5 \times (9 + 4)</td>
</tr>
<tr>
<td>102</td>
<td>42 \times 200</td>
</tr>
<tr>
<td>8400</td>
<td>5 \times 13</td>
</tr>
<tr>
<td></td>
<td>65</td>
</tr>
</tbody>
</table>

**NOTE:** It is not necessary to include exponents since the expressions should be no more complex than the expressions one finds in an application of the associative or distributive property.

**Assessment limit**
- Expressions may contain whole numbers and up to one fraction with a denominator of 10 or less.
- Items may not require division with fractions.
- Items may not contain nested grouping symbols.

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation “add 8 and 7, then multiply by 2” as 2 \times (8 + 7). Recognize that 3 \times (18932 + 921) as three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

<table>
<thead>
<tr>
<th>Students will:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• <strong>apply</strong> an understanding of grouping symbols to <strong>write</strong> numerical expressions.</td>
</tr>
<tr>
<td>• <strong>apply</strong> an understanding of grouping symbols to <strong>interpret</strong> the meaning of numerical expressions without evaluating (i.e., calculating) them.</td>
</tr>
</tbody>
</table>

**Assessment limit**
- Expressions may contain whole numbers or fractions with a denominator of 10 or less.
- Expressions may not include nested parentheses.
- Multiplication cross symbol is the only acceptable symbol for multiplication. The multiplication dot may not be used.
- When grouping symbols are part of the expression, the associative property or distributive property must be found in the expression.
### Aspects of Rigor targeted by the standards in this topic:

- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

### Applicable information from the progression document:

As preparation for the Expressions and Equations Progression in the middle grades, students in Grade 5 begin working more formally with expressions (5.OA.1, 5.OA.2). They write expressions to express a calculation, e.g., writing $2 \times (8+7)$ to express the calculation “add 8 and 7, then multiply by 2.” They also evaluate and interpret expressions, e.g., using their conceptual understanding of multiplication to interpret $3 \times (18932 + 921)$ as being three times as large as $18932 + 921$, without having to calculate the indicated sum or product. Thus, students in Grade 5 begin to think about numerical expressions in ways that prefigure their later work with variable expressions (e.g., three times an unknown length is $3 \times L$). In Grade 5, this work should be viewed as exploratory rather than for attaining mastery; for example, expressions should not contain nested grouping symbols, and they should be no more complex than the expressions one finds in an application of the associative or distributive property, e.g., $(8 + 27) + 2$ or $(6 \times 30) + (6 \times 7)$. Note however that the numbers in expressions need not always be whole numbers. (See p. 32 in the OA Progressions.)
**Topic 14: Graph Points on the Coordinate Plane**

**Standards**

Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).

**Academic Language**

- axes
- coordinate grid/coordinate plane
- coordinates
- corresponding terms
- horizontal
- plot
- ordered pairs
- origin
- point
- quadrant
- vertical
- x-axis
- y-axis
- x-coordinate
- y-coordinate

**Students will:**

- **identify** the intersection of a coordinate plane as the origin and the point where 0 lies on each of the number lines.
- **label** the horizontal axis of a coordinate plane as the x-axis, and the vertical axis as the y-axis.
- **explain** that when plotting points on a coordinate plane the first number in an ordered pair (the x-coordinate) indicates how far to travel from the origin in the direction of the x-axis and that the second number (y-coordinate) indicates how far to travel in the direction of the y-axis.

E.g., When graphing the ordered pair (3,2), 3 is the x-coordinate so you move 3 units from 0 on the x-axis and then, since 2 is the y-coordinate, you move up 2 units in the direction of the y-axis.

**NOTE:** Students are only expected to utilize the first quadrant of the coordinate plane which includes only positive numbers.

**Assessment limit**

- Items may not require directions between two given points. Points must rely on the origin.
- Items may require identifying the point (e.g., Point A) on a coordinate grid that represents a given ordered pair.
- Items may require determining the ordered pair that represents a given point on the coordinate plane.
- Items may not require graphing/plotting a point given an ordered pair.
- Points may only contain positive, whole number ordered pairs.

Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

**MAFS.5.G.1.2**

**Students will:**

- **represent** real world and mathematical problems by graphing points in the first quadrant of the coordinate plane.

  E.g., Students plan to draw a figure in which they plot coordinates that are then connected by line segments.

- **interpret** coordinate values of points in the context of the situation.

**Assessment limit**

Mathematical and real-world problems must have axes scaled to whole numbers (not letters).
Aspects of Rigor targeted by the standards in this topic:

- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.

- **Application:** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

**Applicable information from the progression document:**

Thus, spatial structuring underlies coordinates for the plane as well, and students learn both to apply it and to distinguish the objects that are structured. For example, they learn to interpret the components of a rectangular grid structure as line segments or lines (rather than regions) and understand the precision of location that these lines require, rather than treating them as fuzzy boundaries or indicators of intervals. Students learn to reconstruct the levels of counting and quantification that they had already constructed in the domain of discrete objects to the coordination of (at first) two continuous linear measures. That is, they learn to apply their knowledge of number and length to the order and distance relationships of a coordinate grid and to coordinate this across two dimensions (5.G.1). Although students can often "locate a point," these understandings are beyond simple skills. For example, initially, students often fail to distinguish between two different ways of viewing the point (2, 3), say, as instructions: "right 2, up 3"; and as the point defined by being a distance 2 from the y-axis and a distance 3 from the x-axis. In these two descriptions the 2 is first associated with the x-axis, then with the y-axis. They connect ordered pairs of (whole number) coordinates to points on the grid, so that these coordinate pairs constitute numerical objects and ultimately can be operated upon as single mathematical entities. Students solve mathematical and real-world problems using coordinates. For example, they plan to draw a symmetric figure using computer software in which students' input coordinates that are then connected by line segments (5.G.2).

(See. p. 17 in the G Progressions.)
### Topic 15: Algebra: Analyze Patterns and Relationships

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.</td>
<td>MAFS.5.OA.2.3</td>
</tr>
</tbody>
</table>

**Students will:**
- **generate** two numerical patterns using two given rules.
- **explain** the relationship between the two numerical patterns by comparing the relationship between each of the corresponding terms from each pattern.
- **form** ordered pairs out of corresponding terms from each pattern.
- **graph** the ordered pairs on a coordinate plane.

**Assessment limit**
Expressions may contain whole numbers or fractions with a denominator of 10 or less. Ordered pairs may only be located within Quadrant I of the coordinate plane. Operations in rules limited to: addition, subtraction, multiplication, and division. Patterns that require division may not lead to fractional terms. Items may not contain rules that exceed two procedural operations. Items must provide the rule. Expressions may not include nested parentheses.

**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
- **Procedural skill and fluency:** The Standards call for speed and accuracy in calculation. Students are given opportunities to practice core functions so that they have access to more complex concepts and procedures.
- **Application:** The standards call for students to use math in situations that require mathematical knowledge. Correctly applying mathematical knowledge depends on students having a solid conceptual understanding and procedural fluency.

**Applicable information from the progression document**
Students extend their Grade 4 pattern work by working briefly with two numerical patterns that can be related and examining these relationships within sequences of ordered pairs and in the graphs in the first quadrant of the coordinate plane.5.OA.3. This work prepares students for studying proportional relationships and functions in middle school. (See p. 32 in the OA Progressions.)
### Topic 16: Geometric Measurement Classify Two-Dimensional Figures

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Understand that attributes belonging to a category of two-dimensional</td>
<td>MAFS.5.G.2.3</td>
</tr>
<tr>
<td>figures also belong to all subcategories of that category. For example,</td>
<td>defining attribute</td>
</tr>
<tr>
<td>all rectangles have four right angles and squares are rectangles, so all</td>
<td>category congruent</td>
</tr>
<tr>
<td>squares have four right angles.</td>
<td>parallel perpendicular</td>
</tr>
</tbody>
</table>

**Students will:**

- **categorize** two-dimensional figures (i.e., triangle, quadrilateral, rectangle, square, rhombus, trapezoid) according to their defined attributes.
  
  **NOTE:** Geometric (defining) attributes include number and properties of sides (i.e., parallel, perpendicular, congruent) and number and properties of angles (i.e., type of angle, measurement of angle, congruency of angles).

- **relate** certain categories of two-dimensional figures as subcategories of other categories
  
  E.g., rectangles are a subcategory of parallelograms because they are both quadrilaterals with two pairs of sides that are parallel and congruent, therefore all rectangles are also parallelograms.

- **explain** that the attributes belonging to one category of two-dimensional figures also belong to all subcategories of that category.
  
  E.g., Squares are a subcategory of rectangles, therefore if rectangles have four right angles and two pairs of sides that are parallel and congruent, it can be inferred that squares have four right angles and two pairs of sides that are parallel and congruent.

**Assessment limit**

Attributes of figures may be given or presented within given graphics.

Items that include trapezoids must consider both the inclusive and exclusive definitions.

Items may not use the term "kite" but may include the figure.
Classify and organize two-dimensional figures into Venn diagrams based on the attributes of the figures.  

**NOTE:** This standard has been amended in Florida to include Venn diagrams.

**MAFS.5.G.2.4**

**Students will:**
- **classify** two-dimensional figures (i.e., triangle, quadrilateral, rectangle, square, rhombus, trapezoid) based on defining attributes.
- **organize** two-dimensional figures into a Venn diagram based on defining attributes.

E.g.,

![Venn diagram of quadrilaterals]

**NOTE:** Use the exclusive definition of trapezoid: **exactly** one set of parallel sides.

**NOTE:** Use the inclusive definition of isosceles triangle: triangles with **at least** two sides of the same length

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**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
<table>
<thead>
<tr>
<th>Students learn to analyze and relate categories of two-dimensional and three-dimensional shapes explicitly based on their properties. 5.G.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Based on analysis of properties, they classify two-dimensional figures in hierarchies. For example, they conclude that all rectangles are parallelograms, because they are all quadrilaterals with two pairs of opposite, parallel, equal-length sides (MP3). In this way, they relate certain categories of shapes as subclasses of other categories. 5.G.3</td>
</tr>
<tr>
<td>This leads to understanding propagation of properties; for example, students understand that squares possess all properties of rhombuses and of rectangles. Therefore, if they then show that rhombuses’ diagonals are perpendicular bisectors of one another, they infer that squares’ diagonals are perpendicular bisectors of one another as well.</td>
</tr>
</tbody>
</table>
**Topic 17: Revisiting problem solving with fractions**

This is a culminating topic in which students apply their conceptual understanding from Grades 3-4 and previous Grade 5 topics to a variety of non-routine problem solving situations involving grade-level appropriate operations with fractions. All standards in this topic have been addressed in prior topics. In grade 6, students will finalize their exploration of fraction operations with dividing fractions by fractions using visual models and equations.

<table>
<thead>
<tr>
<th>Standards</th>
<th>Academic Language</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{2}{7} &lt; \frac{1}{2}$.</td>
<td>MAFS.5.NF.1.2</td>
</tr>
</tbody>
</table>

**Students will:**
- solve word problems involving addition and subtraction of fractions with like and unlike denominators.
- use benchmark fractions and number sense of fractions to estimate and assess reasonableness of answers.

**NOTE:** See the Common Addition and Subtraction Situations Table for examples of word problem types (p.47).

| Solve real world problems involving multiplication of fractions and mixed numbers, e.g. by using visual fraction models or equations to represent the problem. | MAFS.5.NF.2.6 |

**Students will:**
- solve real world problems involving multiplication of fractions and mixed numbers (including multiplying mixed numbers by mixed numbers).

| Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. (Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.) c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share $\frac{1}{2}$-lb of chocolate equally? How many $\frac{1}{3}$-cup servings are in 2 cups of raisins? | MAFS.5.NF.2.7 |

**Students will:**
- solve real world problems involving division of unit fractions and whole numbers using fraction models and equations.

**NOTE:** Division of a fraction by a fraction is a Grade 6 standard.

**NOTE:** Allow opportunities for students to create context for whole number divided by unit fraction and unit fraction divided by whole number.

**Aspects of Rigor targeted by the standards in this topic:**
- **Conceptual understanding:** The Standards call for conceptual understanding of key concepts. Students must be able to access concepts from a number of perspectives so that they are able to see math as more than a set of mnemonics or discrete procedures.
Critical Areas for Mathematics in Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (i.e., unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.
# Grade 5 Major, Supporting, and Additional Work

<table>
<thead>
<tr>
<th>Topic</th>
<th>Title</th>
<th>Major Work</th>
<th>Supporting Work</th>
<th>Additional Work</th>
</tr>
</thead>
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<td>1</td>
<td>Understanding Place Value</td>
<td>5.NBT.1.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5.NBT.1.2</td>
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<td>5.NBT.1.3</td>
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<td></td>
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<td>5.NBT.1.4</td>
<td></td>
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<tr>
<td>2</td>
<td>Use Models and Strategies to Add and Subtract Decimals</td>
<td>5.NBT.2.7</td>
<td></td>
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</tr>
<tr>
<td>3</td>
<td>Fluently Multiply Multi-Digit Whole Numbers</td>
<td>5.NBT.2.5</td>
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<tr>
<td>4</td>
<td>Use Models and Strategies to Multiply Decimals</td>
<td>5.NBT.2.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Use Models and Strategies to Divide Whole Numbers</td>
<td>5.NBT.2.6</td>
<td></td>
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<tr>
<td>6</td>
<td>Use Models and Strategies to Divide Decimals</td>
<td>5.NBT.2.7</td>
<td></td>
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</tr>
<tr>
<td>7</td>
<td>Use Equivalent Fractions to Add and Subtract Fractions</td>
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<td>5.NF.1.2</td>
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<tr>
<td>8</td>
<td>Apply Understanding of Multiplication to Multiply Fractions</td>
<td>5.NF.2.4</td>
<td></td>
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<td></td>
<td></td>
<td>5.NF.2.5</td>
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<td>5.NF.2.6</td>
<td></td>
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</tr>
<tr>
<td>9</td>
<td>Apply Understanding of Division to Divide Fractions</td>
<td>5.NF.2.3</td>
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<td></td>
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<td>5.NF.2.7</td>
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<tr>
<td>10</td>
<td>Represent and Interpret Data</td>
<td></td>
<td>5.MD.2.2</td>
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<td>11</td>
<td>Understand Volume Concepts</td>
<td>5.MD.3.3</td>
<td></td>
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<td>5.MD.3.5</td>
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<td>12</td>
<td>Convert Measurements</td>
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<td></td>
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<td>5.OA.1.2</td>
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<td>5.NF.1.2</td>
<td>5.NF.2.6</td>
<td>5.NF.2.7 (c)</td>
</tr>
</tbody>
</table>
Standards for Mathematical Practice

Grade 5 students will:

1. Make sense of problems and persevere in solving them. (SMP.1)
   Mathematically proficient students in Grade 5 solve problems by applying their understanding of operations with whole numbers, decimals, and fractions including mixed numbers. They solve problems related to volume and measurement conversions. Students seek the meaning of a problem and look for efficient ways to represent and solve it. They may check their thinking by asking themselves, “What is the most efficient way to solve the problem?” “Does this make sense?”

2. Reason abstractly and quantitatively. (SMP.2)
   Mathematically proficient students in Grade 5 recognize that a number represents a specific quantity. They extend this understanding from whole numbers to work with fractions and decimals. This involves two processes—decontextualizing and contextualizing. Grade 5 students decontextualize by taking a real-world problem and writing and solving equations based on the word problem. For example, consider the task, “There are 2 2/3 of a yard of rope in the shed. If a total of 4 1/6 yard is needed for a project, how much more rope is needed?” Students decontextualize the problem by writing the equation 4 1/6 – 2 2/3 = ___ and then solving it. Further, students contextualize the problem after they find the answer, by reasoning that 1 3/6 or 1 ½ yards of rope is the amount needed. Further, Grade 5 students write simple expressions that record calculations with numbers and represent or round numbers using place value concepts.

3. Construct viable arguments and critique the reasoning of others. (SMP.3)
   Mathematically proficient students in Grade 5 construct arguments using representations, such as objects, pictures, and drawings. They explain calculations based upon models and properties of operations and rules that generate patterns. They demonstrate and explain the relationship between volume and multiplication. They refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” They explain their thinking to others and respond to others’ thinking through discussions or written responses.

4. Model with mathematics. (SMP.4)
   Mathematically proficient students in Grade 5 experiment with representing problem situations in multiple ways including numbers, words (mathematical language), drawing pictures, using objects, making a chart, list, or graph, creating equations, etc. Students need opportunities to connect the different representations and explain the connections. They should be able to use all of these representations as needed. Fifth graders should evaluate their results in the context of the situation and whether the results make sense. They also evaluate the utility of models to determine which models are most useful and efficient to solve problems.

5. Use appropriate tools strategically. (SMP.5)
   Mathematically proficient students in Grade 5 consider the available tools (including estimation) when solving a mathematical problem and decide when certain tools might be helpful. For instance, they may use unit cubes to fill a rectangular prism and then use a ruler to measure the dimensions. They use graph paper to accurately create graphs and solve problems or make predictions from real world data.

6. Attend to precision. (SMP.6)
   Mathematically proficient students in Grade 5 continue to refine their mathematical communication skills by using clear and precise language in their discussions with others and in their own reasoning. Students use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. They are careful about specifying units of measure and state the meaning of the symbols they choose. For instance, when figuring out the volume of a rectangular prism they record their answers in cubic units.

7. Look for and make use of structure. (SMP.7)
   Mathematically proficient students in Grade 5 look closely to discover a pattern or structure. For instance, students use properties of operations as strategies to add, subtract, multiply and divide with whole numbers, fractions, and decimals. They examine numerical patterns and relate them to a rule or a graphical representation.

8. Look for and express regularity in repeated reasoning. (SMP.8)
   Mathematically proficient students in Grade 5 use repeated reasoning to understand algorithms and make generalizations about patterns. Students connect place value and their prior work with operations to understand algorithms to fluently multiply multi-digit numbers and perform all operations with decimals to hundredths. Students explore operations with fractions with visual models and begin to formulate generalizations.
## Common Addition and Subtraction Situations Table

<table>
<thead>
<tr>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Add to</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?</td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two?</td>
<td>Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?</td>
</tr>
<tr>
<td>$2 + 3 = ?$</td>
<td>$2 + ? = 5$</td>
<td>$? + 3 = 5$</td>
</tr>
<tr>
<td><strong>Take from</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now?</td>
<td>Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before?</td>
</tr>
<tr>
<td>$5 - 2 = ?$</td>
<td>$5 - ? = 3$</td>
<td>$? - 2 = 3$</td>
</tr>
<tr>
<td><strong>Total Unknown</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Three red apples and two green apples are on the table. How many apples are on the table?</td>
<td>Grandma has five flowers. How many can she put in her red vase and how many in her blue vase?</td>
<td>Five apples are on the table. Three are red and the rest are green. How many apples are green?</td>
</tr>
<tr>
<td></td>
<td>$5 = 0 + 5$, $5 = 5 + 0$</td>
<td>$5 - 3 = ?$</td>
</tr>
<tr>
<td></td>
<td>$5 = 1 + 4$, $5 + 4 + 1$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$5 = 2 + 3$, $5 = 3 + 2$</td>
<td></td>
</tr>
<tr>
<td><strong>Difference Unknown</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>“How many more?” version:</td>
<td>“More” version suggests operation:</td>
<td>“Fewer” version suggests operation:</td>
</tr>
<tr>
<td>Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?</td>
<td>Julie has 3 more apples than Lucy. Lucy has two apples. How many apples does Julie have?</td>
<td>Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>$2 + ? = 5$</td>
<td>$2 + 3 = ?$</td>
<td>$5 - 3 = ?$</td>
</tr>
<tr>
<td>$5 - 2 = ?$</td>
<td>$3 + 2 = ?$</td>
<td>$? + 3 = 5$</td>
</tr>
<tr>
<td>“How many fewer?” version:</td>
<td>“Fewer” version suggests wrong operation:</td>
<td>“More” version suggests wrong operation:</td>
</tr>
<tr>
<td>Lucy has two apples. Julie has five apples. How may fewer apples does Lucy have than Julie?</td>
<td>Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have?</td>
<td>Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have?</td>
</tr>
<tr>
<td>$2 + ? = 5$</td>
<td>$2 + 3 = ?$</td>
<td>$5 - 3 = ?$</td>
</tr>
<tr>
<td></td>
<td>$3 + 2 = ?$</td>
<td>$? + 3 = 5$</td>
</tr>
</tbody>
</table>

Darker shading indicates the four Kindergarten problem subtypes. Grade 1 and 2 students work with all subtypes and variants. Unshaded (white) problems are the four difficult subtypes or variants that students should work with in Grade 1 but need not master until Grade 2. Adapted from CCSS, p. 88, which is based on Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity, National Research Council, 2009, pp. 32–33.

1 This can be used to show all decompositions of a given number, especially important for numbers within 10. Equations with totals on the left help children understand that = does not always mean “makes” or “results in” but always means “is the same number as.” Such problems are not a problem subtype with one unknown, as is the Addend Unknown subtype to the right. These problems are a productive variation with two unknowns that give experience with finding all the decompositions of a number and reflecting on the patterns involved.
### Common Multiplication and Division Situations Table

<table>
<thead>
<tr>
<th>Equal Groups</th>
<th>Arrays, Area</th>
<th>Compare</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unknown Product</strong></td>
<td><strong>Unknown Factor</strong></td>
<td><strong>Multiplier Unknown</strong></td>
</tr>
<tr>
<td>$3 \times 6 = ?$</td>
<td>$\frac{1}{3} \times 18 = ?$</td>
<td>$\times \frac{1}{3} = 6$</td>
</tr>
<tr>
<td><strong>Group Size Unknown</strong></td>
<td><strong>Number of Groups Unknown</strong></td>
<td></td>
</tr>
<tr>
<td>(“How many in each group?” Division)</td>
<td>(“How many groups?” Division)</td>
<td></td>
</tr>
<tr>
<td>$3 \times ? = 18$ and $18 \div 3 = ?$</td>
<td>$\times 6 = 18$ and $18 \div 6 = ?$</td>
<td></td>
</tr>
<tr>
<td><strong>Measurement example.</strong> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</td>
<td><strong>Measurement example.</strong> You have 18 plums to be packed 6 to a bag, then how many bags are needed?</td>
<td><strong>Measurement example.</strong> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</td>
</tr>
</tbody>
</table>

1. The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.
2. The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many pieces of string will you have?
3. Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.
4. Multiplicative Compare problems appear first in Grade 4, with whole-number values for A, B, and C, and with the “times as much” language in the table. In Grade 5, unit fractions language such as “one third as much” may be used. Multiplying and unit fraction language change the subject of the comparing sentence, e.g., “A red hat costs $A$ times as much as the blue hat” results in the same comparison as “A blue hat costs $\frac{1}{A}$ times as much as the red hat,” but has a different subject.
Grade 5 FSA Mathematics Reference Sheet

Customary Conversions

1 foot = 12 inches
1 yard = 3 feet
1 mile = 5,280 feet
1 mile = 1,760 yards

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Metric Conversions

1 meter = 100 centimeters
1 meter = 1000 millimeters
1 kilometer = 1000 meters

1 liter = 1000 milliliters

1 gram = 1000 milligrams
1 kilogram = 1000 grams

Time Conversions

1 minute = 60 seconds
1 hour = 60 minutes
1 day = 24 hours
1 year = 365 days
1 year = 52 weeks